Lecture 2: 1.3-1.5

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Composition of functions (1.3)

Basic example: f(x)=3x+2

g(y)=2y

(f∘g)(x) = f(g(y)) = f(2y) = 3*(2y)+2 = **6y+2**

<u>Ex</u>: $f(x) = 3x, g(x) = x^2$

(1) $(f \circ g)(x) = f\big(g(x)\big) = f(x^2) = 3 * x^2$

(2) $(g \circ f)(x) = g\big(f(x)\big) = g(3x) = (3x)^2 = 9x^2$

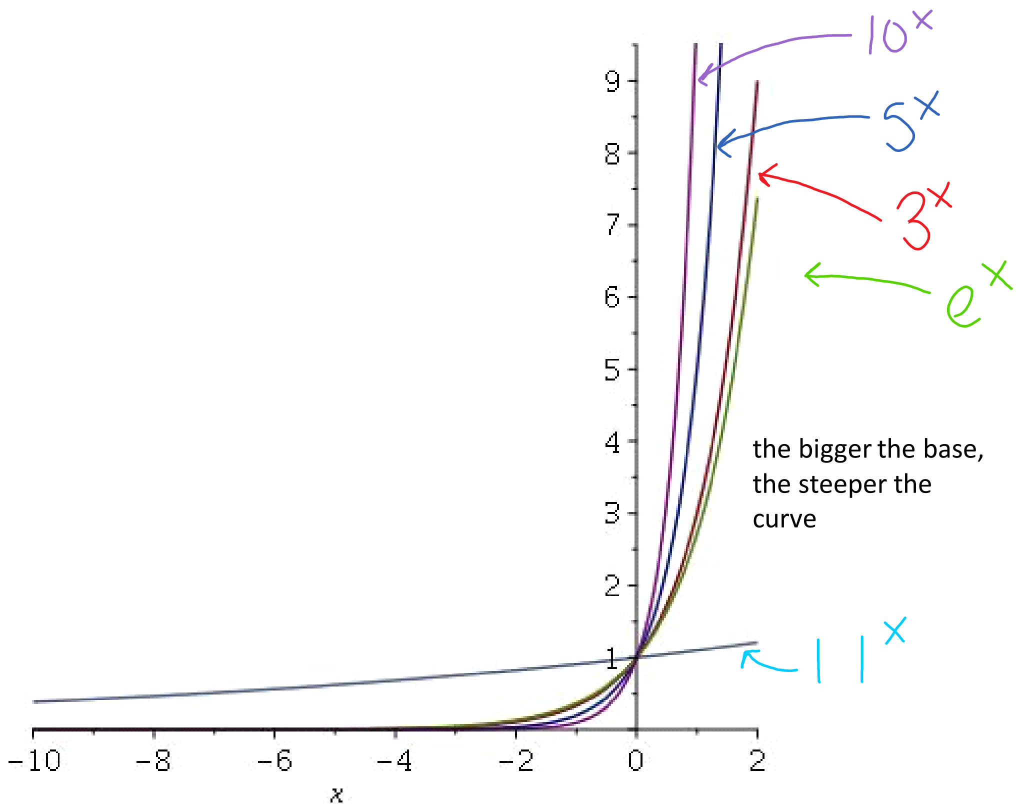
--> Order matters!

$\boldsymbol{(f \circ g)(x) \neq (g \circ f)(x)}$

Exponential functions (1.4)

Eg: $f(x) = e^x$

In general: $f(x) = a^x, a \epsilon \mathbb{R}$



the bigger the base, the steeper the curve

Always goes through (0,1). Why? $a^0 = 1, a\epsilon\mathbb{R}$

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Power rules

$$(a^b)^c = a^{bc} \qquad a^0 = 1$$

$$a^{-b} = \frac{1}{a^b} \qquad a^1 = a$$

$$a^b a^c = a^{b+c} \Big/ \frac{a^b}{a^c} = a^{b-c}$$

$$(ab)^c = a^c b^c$$

Inversion functions and logarithms (1.5)

Recall: $(f \circ g)(x) = f(g(x))$

An inverse function (f^{-1}) of f is such that $(f^{-1} \circ f)(x) = 1$ & $(f \circ f^{-1})(x) = 1$

not every function has an inverse

Inverse: given a y in R, what is x?

Things we know: if I tell you $x^3 = 8$, you know that x=2.

Cubic power <--> cubic root

inverse!

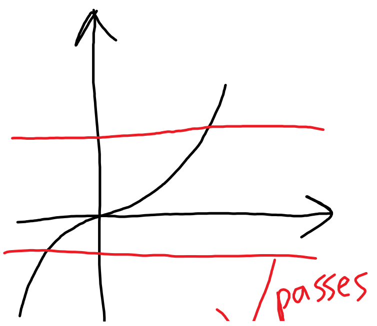
Ex. $x^2 = 4$

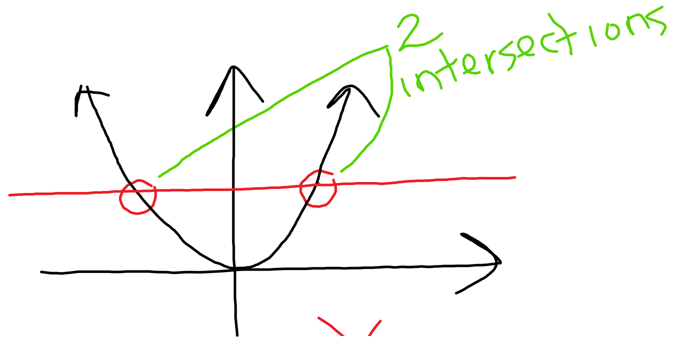
2 answers: +-2 → inverse not unique on all real #s!

We can define the inverse function again only a specific domain and range.

One-to-one function: if and only if no horizontal line intersects the graph more than once

-> horizontal line test



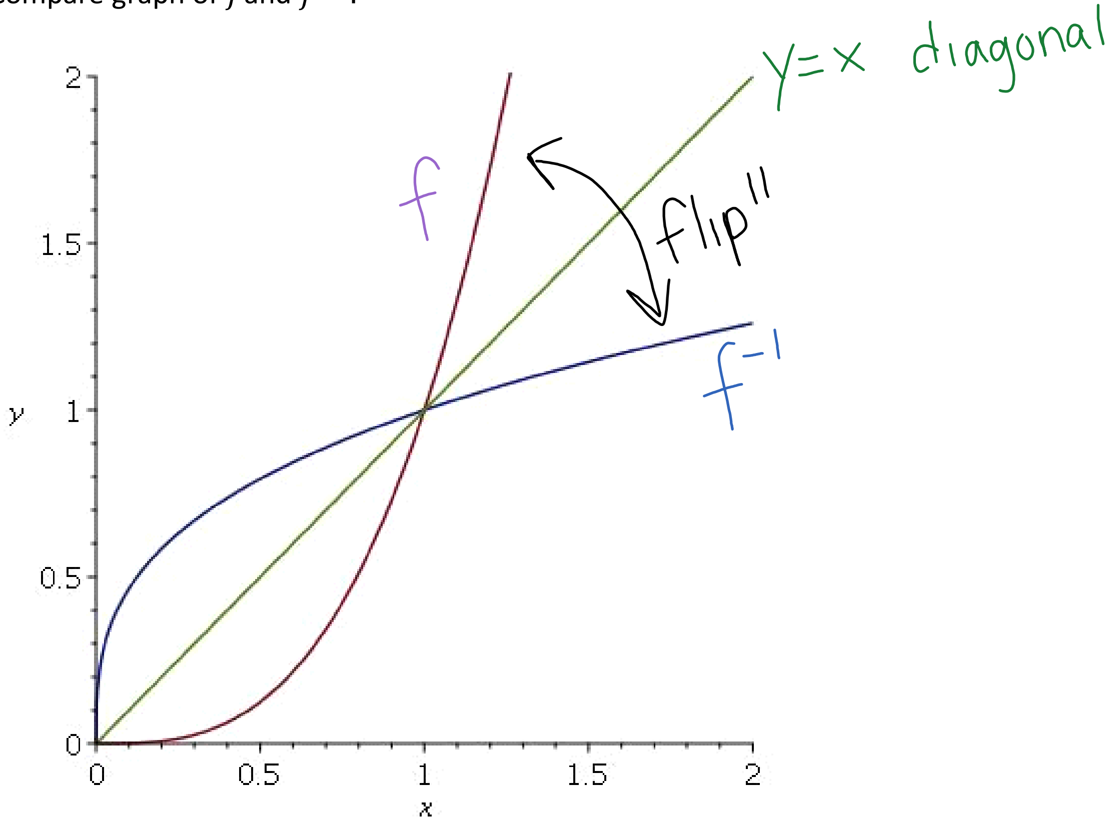


Let f be a one-to-one function with domain A and range B. Then it's inverse function f^{-1} has domain B and range A and is defined by:

$$f^{-1}(y) = x \iff f(x) = y$$ for any y in B

if and only if

Compare graph of f and f^{-1}:



Logarithmic Functions --> inverse of exponential functions

b^y — exponent, base | $\log_b x$ — argument, base

$$\log_b x = y \iff b^y = x$$

Rules

$$\log_b(b) = 1 \quad \text{eg } \ln(e) = 1$$

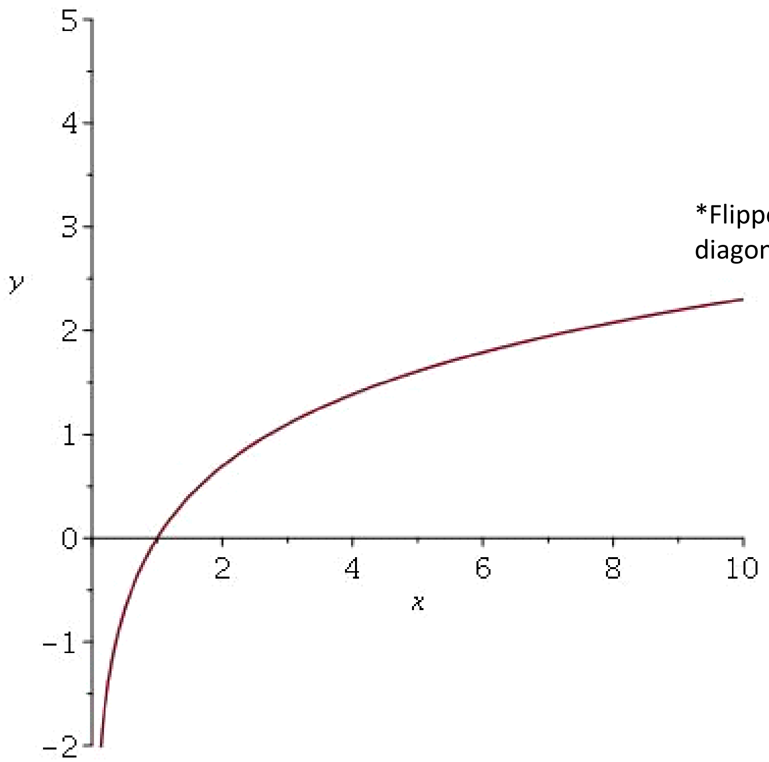
$$\log_b(b^x) = x$$

$$b^{\log_b(x)} = x$$

$$\log(xy) = \log(x) + \log(y)$$

$$\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$

$$\log_b(x^r) = r \log_b(x)$$



Flipped exponential function graph along y=x diagonal because they're inverses

Ex. Solve for x:

$$\ln(x^2 - 1) = 3 \quad \text{/raise to power of e}$$

$$e^{\ln(x^2-1)} = e^3$$

$\ln(x^2-1)=3$ /raise to power of e

$e^{\ln(x^2-1)} = e^3$

$x^2-1=e^3$ /+1 on both sides

$x^2=e^3+1$

$x=\pm\sqrt{e^3+1}$ /2 answers

<u>Ex.</u> Solve for x:

$\ln(x)+\ln(x-1)=2$

$\ln\big(x(x-1)\big)=1$

$e^{\ln(x^2-x)}=e$ /raise both sides to e

$x^2-x=e$

$x^2-x-e=0$ /plug into quadratic formula

$x_{1,2}=\dfrac{1\pm\sqrt{1+4e}}{2}$